Non-Gaussian GARCH option pricing models and their diffusion limits

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Abstract

We investigate the weak convergence of a non-Gaussian GARCH model together with an application to the pricing of European style options determined using two different stochastic discount factors: the extended Girsanov principle of Elliott and Madan (1998) and the conditional Esscher transform. Applying these changes of measure to asymmetric GARCH models sampled at increasing frequencies, we obtain two risk neutral families of processes which converge to the same bivariate diffusion, which is no longer a standard Hull-White stochastic volatility process. Additionally, it differs from the one obtained by applying the standard minimal martingale measure in continuous time to the diffusion limit of the GARCH family of processes under the physical measure. We show that for skewed innovations, this risk neutral diffusion limit exhibits a non-zero market price of volatility risk which is proportional to the market price of the equity risk, where the constant of proportionality depends on the skewness and kurtosis of the underlying distribution. Our theoretical results are further supported by extensive simulations.

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