On the Depletion Problem for an Insurance Risk Process: New Non-ruin Quantities in Collective Risk Theory

Zied Ben-Salah [*]
University of Montreal

Hélène Guérin [†]
University of Rennes

Manuel Morales [‡]
University of Montreal

Hassan Omidi Firouzi [§]
University of Montreal

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Abstract

The field of risk theory has traditionally focused on ruin-related quantities. In particular, the so-called Expected Discounted Penalty Function has been the object of a thorough study over the years. Although interesting in their own right, ruin related quantities do not seem to capture path-dependent properties of the reserve. In this article we aim at presenting the probabilistic properties of drawdowns and the speed at which an insurance reserve depletes as a consequence of the risk exposure of the company. These new quantities are not ruin related yet they capture important features of an insurance position and we believe it can lead to the design of a meaningful risk measures. Studying drawdowns and speed of depletion for Lévy insurance risk processes represent a novel and challenging concept in insurance mathematics. Indeed, drawdowns and speed of depletion are quantities that do not depend on the level but rather on path properties of the model which explain how fast the process can drop. This type of quantities has never been proposed before as measures of riskiness in insurance. Drawdowns have been only studied for diffusion processes in a finance setting, yet in insurance we need expressions for processes exhibiting jumps. In this joint work, we give all these concepts are formally introduced in an insurance setting. Moreover, using recent results in fluctuation theory for Lévy processes, we derive expressions for the distribution of several quantities related to the depletion problem. Of particular interest are the distribution of drawdowns and the Laplace transform for the speed of depletion. These expressions are given for some examples of Lévy insurance risk processes for which they can be calculated, in particular for the classical Cramer-Lundberg model.

[*]University of Montreal, CANADA. Email: bensalah@dms.umontreal.ca

[†]University of Rennes, FRANCE. Email: helene.guerin@univ-rennes1.fr

[‡]Department of Mathematics and Statistics. University of Montreal. CP. 6128 succ. centre-ville. Montreal, Quebec. H3C 3J7. CANADA. Email: morales@dms.umontreal.ca

[§]Hassan Omidi Firouzi. Department of Mathematics and Statistics. University of Montreal. CP. 6128 succ. centre-ville. Montreal, Quebec. H3C 3J7. CANADA. Email: omidifh@dms.umontreal.ca